n° 93 p 182

$$I = \int_0^{\pi/2} \frac{\cos x}{1+2\sin x}\, dx \qquad J = \int_0^{\pi/2} \frac{\sin 2x}{1+2\sin x}\, dx$$

1 - $\forall x \in \mathbb{R} \quad f(x) = \frac{\cos x}{1+2\sin x}$.

$f = \frac{1}{2}\frac{u'}{u}$ avec $\forall x \in [0;\frac{\pi}{2}]\ u(x) > 0$

$F = \frac{1}{2} \ln u$

$F(x) = \frac{1}{2} \ln(1+2\sin x)$

$I = F\left(\frac{\pi}{2}\right) - F(0)$

$I = \frac{1}{2}\ln\left(1+2\sin\frac{\pi}{2}\right) - \frac{1}{2}\ln(1+2\sin 0)$

$I = \frac{1}{2}\ln 3$

$I = \frac{1}{2}\ln 3$

2. $I + J = \int_0^{\pi/2} \frac{\cos x}{1+2\sin x}\, dx + \int_0^{\pi/2} \frac{\sin 2x}{1+2\sin x}\, dx$

$I + J = \int_0^{\pi/2} \frac{\cos x + \sin 2x}{1+2\sin x}\, dx$

$\forall x \in \mathbb{R} \quad \sin(2x) = 2\sin x \cos x$

$I + J = \int_0^{\pi/2} \frac{\cos x\,(1+2\sin x)}{1+2\sin x}\, dx$

$I + J = \int_0^{\pi/2} \cos x \;\, dx$

$\forall x \in \mathbb{R} \quad g(x) = \cos x$

$G(x) = \sin x$

$I + J = G\left(\frac{\pi}{2}\right) - G(0)$

$I + J = 1$

3 - $\begin{cases} I = \frac{1}{2}\ln 3 \\ I + J = 1 \end{cases} \Leftrightarrow \begin{cases} I = \frac{1}{2}\ln 3 \\ J = 1 - \frac{1}{2}\ln 3 \end{cases}$