n° 92 p 182

$$I = \int_0^1 \frac{e^{2x}}{1+e^{2x}}\,dx \qquad J = \int_0^1 \frac{1}{1+e^{2x}}\,dx$$

1. $\forall x \in [0;1]$ $\quad f(x) = \dfrac{e^{2x}}{1+e^{2x}}$

$f = \dfrac{1}{2}\,\dfrac{u'}{u}$ avec $u > 0$

donc $F = \dfrac{1}{2}\ln u$

$F(x) = \dfrac{1}{2}\ln(1+e^{2x})$

donc : $I = F(1) - F(0)$

$I = \dfrac{1}{2}\ln(1+e^{2\times 1}) - \dfrac{1}{2}\ln(1+e^{0})$

$I = \dfrac{1}{2}(\ln 1 + e^2) - \dfrac{1}{2}\ln 2$

$I = \dfrac{1}{2}\ln\dfrac{1+e^2}{2}$

$I + J = \displaystyle\int_0^1 \frac{e^{2x}}{1+e^{2x}}\,dx + \int_0^1 \frac{1}{1+e^{2x}}\,dx$

$I + J = \displaystyle\int_0^1 \frac{e^{2x}+1}{e^{2x}+1}\,dx$

$I + J = \displaystyle\int_0^1 1\,dx$

$\forall x \in [0;1]$ $\quad g(x) = 1$

$G(x) = x$

$I + J = G(1) - G(0)$

$I + J = 1$

$$\begin{cases} I = \dfrac{1}{2}\ln\dfrac{1+e^2}{2} \\ I + J = 1 \end{cases} \iff \begin{cases} I = \dfrac{1}{2}\ln\dfrac{1+e^2}{2} \\ J = 1 - \dfrac{1}{2}\ln\dfrac{1+e^2}{2} \end{cases}$$